Topic: All possible values for the angle theta

Question: Which of the following is a set of angles, in radians, that satisfy the equation?

$$\sin\theta = \sin 60^\circ$$

Answer choices:

A $\left\{-\frac{2\pi}{3} \pm 2\pi k:\ k = 0,1,2,\ldots\right\}$

B $\left\{\frac{\pi}{3} \pm 2\pi k:\ k = 0,1,2,\ldots\right\}$

C $\left\{-\frac{\pi}{3} \pm 2\pi k:\ k = 0,1,2,\ldots\right\}$

D $\left\{\frac{4\pi}{3} \pm 2\pi k:\ k = 0,1,2,\ldots\right\}$

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Solution: B

Recall that for any angle θ, the value of $\sin\theta$ is equal to the y-coordinate of the point at which the terminal side of θ intersects the unit circle. Thus any angle θ that satisfies the equation $\sin\theta = \sin 60^\circ$ must be in a quadrant in which the y-coordinate of every point has the same sign as $\sin 60^\circ$. Note that an angle of 60° is in the first quadrant, a quadrant in which the y-coordinate of every point is positive. Therefore, $\sin 60^\circ$ is positive. By extension, every angle θ that satisfies the equation $\sin\theta = \sin 60^\circ$ must be in a quadrant in which the y-coordinate of every point is positive. Thus every angle that satisfies the equation $\sin\theta = \sin 60^\circ$ is in either the first quadrant or the second quadrant.

In degrees, every angle θ which is in the first quadrant and satisfies the equation $\sin\theta = \sin 60^\circ$ is equal to $60^\circ \pm (360^\circ)k$ for some integer k. Now we will convert angles of this form in degrees to angles in radians, using the fact that $\pi = 180^\circ$:

$$60^\circ = 60^\circ(1)$$

$$60^\circ = 60^\circ\left(\frac{\pi}{180^\circ}\right)$$

$$60^\circ = \frac{60\pi}{180}$$

$$60^\circ = \frac{\pi}{3}$$

Thus the angles (in degrees) in the set

$$\{60^\circ \pm (360^\circ)k:\ k = 0,1,2,\ldots\}$$

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are equivalent to the angles (in radians) in the set

$$\left\{ \frac{\pi}{3} \pm 2\pi k : \ k = 0,1,2,\ldots \right\}$$

This is the set of angles given in answer choice B. We will show that none of the other answer choices is correct.

First, note that an angle of $-2\pi/3$ radians is in the third quadrant, so $\sin(-2\pi/3)$ is negative. Also, $-2\pi/3$ can be expressed as

$$-\frac{2\pi}{3} + 2\pi(0)$$

which is one of the angles included in the set given in answer choice A (namely, the one with $k = 0$). This rules out answer choice A.

Next, note that an angle of $-\pi/3$ radians is in the fourth quadrant, so $\sin(-\pi/3)$ is negative. Also, $-\pi/3$ can be expressed as

$$-\frac{\pi}{3} + 2\pi(0)$$

which is one of the angles included in the set given in answer choice C. This rules out answer choice C.

Finally, note that an angle of $4\pi/3$ radians is in the third quadrant, so $\sin(4\pi/3)$ is negative. Also, $4\pi/3$ can be expressed as

$$\frac{4\pi}{3} + 2\pi(0)$$

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which is one of the angles included in the set given in answer choice D. This rules out answer choice D.

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Topic: All possible values for the angle theta

Question: Which of the following is the set of angles that satisfy the equation?

$$\cos\theta = -0.194$$

Answer choices:

A $\{101^\circ \pm (360^\circ)k : \ k = 0,1,2,\ldots\} \cup \{259^\circ \pm (360^\circ)k : \ k = 0,1,2,\ldots\}$

B $\{79^\circ \pm (360^\circ)k : \ k = 0,1,2,\ldots\} \cup \{-101^\circ \pm (360^\circ)k : \ k = 0,1,2,\ldots\}$

C $\{-101^\circ \pm (360^\circ)k : \ k = 0,1,2,\ldots\} \cup \{79^\circ \pm (360^\circ)k : \ k = 0,1,2,\ldots\}$

D $\{259^\circ \pm (360^\circ)k : \ k = 0,1,2,\ldots\} \cup \{-79^\circ \pm (360^\circ)k : \ k = 0,1,2,\ldots\}$

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Solution: A

Recall that for any angle θ, the value of $\cos\theta$ is equal to the x-coordinate of the point at which the terminal side of θ intersects the unit circle. Thus any angle θ that satisfies the equation $\cos\theta = -0.194$ must be in a quadrant in which the x-coordinate of every point is negative, so it must be in either the second quadrant or the third quadrant. First, we'll find an angle θ in the second quadrant which satisfies the equation. In particular, we'll find an angle in the interval $(90°,180°)$ which satisfies it. To do this, we'll use a calculator and find the "inverse" of the cosine function at -0.194, which gives us an angle of about $101°$.

Next, we'll find an angle θ in the third quadrant whose cosine is equal to -0.194. In particular, we'll find an angle in the interval $(180°,270°)$ which satisfies it. Note that $101 = 180 - 79$, so an angle of $101°$ is $79°$ "above" the negative x-axis. Therefore, by symmetry, the angle which is in the interval $(180°,270°)$ and whose cosine is equal to -0.194 must be $79°$ "below" the negative x-axis. This angle is $(180 + 79)° = 259°$.

Combining these results, we find that the solution set of the equation $\cos\theta = -0.194$ is

$$\{101° \pm (360°)k : \ k = 0,1,2,\ldots\} \cup \{259° \pm (360°)k : \ k = 0,1,2,\ldots\}$$

Thus answer choice A is correct. We will show that each of the other answer choices includes an angle in the first quadrant or an angle in the fourth quadrant. The x-coordinate of any point in the first quadrant is positive, as is the x-coordinate of any point in the fourth quadrant. Therefore, the cosine of any angle in the first quadrant or the fourth quadrant is positive, and such an angle cannot be a solution of the

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equation $\cos\theta = -0.194$. Answer choices B and C include an angle of $79°$, which is in the first quadrant. Answer choice D includes an angle of $-79°$, which is in the fourth quadrant.

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Topic: All possible values for the angle theta

Question: Which of the following is a set of angles (in radians) that satisfy the equation?

$$\cos\theta = \cos\left(\frac{11\pi}{8}\right)$$

where $\frac{11\pi}{8}$ is in radians

Answer choices:

A $\left\{-\frac{57\pi}{8} \pm 2\pi k : \ k = 0,1,2,\ldots\right\}$

B $\left\{\frac{45\pi}{8} \pm 2\pi k : \ k = 0,1,2,\ldots\right\}$

C $\left\{-\frac{75\pi}{8} \pm 2\pi k : \ k = 0,1,2,\ldots\right\}$

D $\left\{\frac{29\pi}{8} \pm 2\pi k : \ k = 0,1,2,\ldots\right\}$

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Solution: C

One possibility is that an angle of $11\pi/8$ is included in one of the given sets of angles. Since any two angles in each of the given sets differ from each other by some multiple of 2π, we can easily determine if this possibility occurs, by comparing the difference between just one angle in each set and an angle of $11\pi/8$. We will compute the differences between the $k = 0$ angle in each set (which we'll call θ) and $11\pi/8$.

For $\theta = -\frac{57\pi}{8}$:

$$\theta - \frac{11\pi}{8} = \frac{(-57 - 11)\pi}{8} = -\frac{68\pi}{8} = -\frac{17\pi}{2}$$

For $\theta = \frac{45\pi}{8}$:

$$\theta - \frac{11\pi}{8} = \frac{(45 - 11)\pi}{8} = \frac{34\pi}{8} = \frac{17\pi}{4}$$

For $\theta = -\frac{75\pi}{8}$:

$$\theta - \frac{11\pi}{8} = \frac{(-75 - 11)\pi}{8} = -\frac{86\pi}{8} = -\frac{43\pi}{4}$$

For $\theta = \frac{29\pi}{8}$:

$$\theta - \frac{11\pi}{8} = \frac{(29 - 11)\pi}{8} = \frac{18\pi}{8} = \frac{9\pi}{4}$$

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None of the results of this subtraction process yields an angle which is an integer multiple of 2π. Therefore, we must use a different approach to determine the answer to the question. We will use symmetry.

Note that

$$\pi = \frac{8\pi}{8} < \frac{(8+3)\pi}{8} = \frac{11\pi}{8} < \frac{12\pi}{8} = \frac{3\pi}{2}$$

This tells us that $11\pi/8$ is in the third quadrant, and in particular that $11\pi/8$ is in the interval $(\pi,3\pi/2)$. Moreover,

$$\frac{11\pi}{8} = \frac{(8+3)\pi}{8} = \frac{8\pi}{8} + \frac{3\pi}{8} = \pi + \frac{3\pi}{8}$$

Therefore, an angle of measure $11\pi/8$ is $3\pi/8$ radians "below" the negative x-axis, so one of the given sets of angles must contain an angle in the interval $(\pi/2,\pi)$ which is $3\pi/8$ radians "above" the negative x-axis. That is, one of the given sets of angles must contain an angle of measure

$$\pi - \frac{3\pi}{8} = \frac{8\pi - 3\pi}{8} = \frac{5\pi}{8}$$

Then the measure of every angle in the same set as the angle of measure $5\pi/8$ would differ from $5\pi/8$ by some integer multiple of 2π.

Here's what we're going to do: We'll subtract $5\pi/8$ from the $k = 0$ angle (which we'll again call θ) in each of the four given sets of angles. Then the correct answer to our question will be the set of angles that contains the angle θ for which $\theta - (5\pi/8)$ is an integer multiple of 2π.

For $\theta = -\frac{57\pi}{8}$:

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$$-\frac{57\pi}{8}-\frac{5\pi}{8}=\frac{(-57-5)\pi}{8}=-\frac{62\pi}{8}=-\frac{31\pi}{4}$$

For $\theta=\frac{45\pi}{8}$:

$$\frac{45\pi}{8}-\frac{5\pi}{8}=\frac{(45-5)\pi}{8}=\frac{40\pi}{8}=5\pi$$

For $\theta=-\frac{75\pi}{8}$:

$$-\frac{75\pi}{8}-\frac{5\pi}{8}=\frac{(-75-5)\pi}{8}=-\frac{80\pi}{8}=-10\pi$$

For $\theta=\frac{29\pi}{8}$:

$$\frac{29\pi}{8}-\frac{5\pi}{8}=\frac{(29-5)\pi}{8}=\frac{24\pi}{8}=3\pi$$

What we have found is that the only subtraction that resulted in an integer multiple of 2π is the one with $\theta=-(75/8)\pi$. Thus the answer to our question is the set of angles that contains an angle of $-(75/8)\pi$, so the correct answer is C.

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